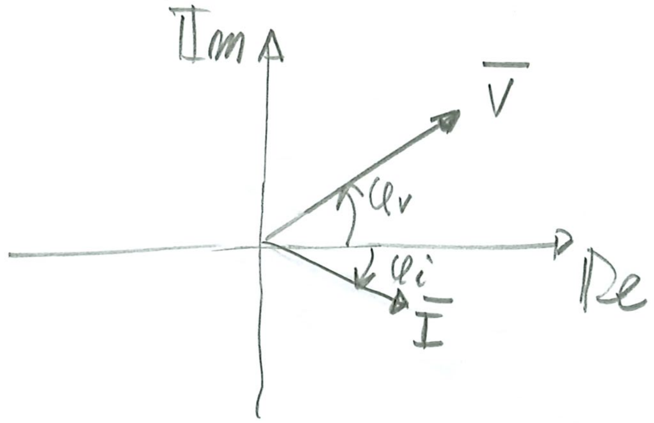
Résumé: **CM # 8 → Circuits AC (Partie II)**

Evolution temporelle → Phasor

$$v(t) = V_m \cdot \sin(\omega t + \varphi_v) \longrightarrow \bar{V} = V_m \angle \varphi_v$$

$$i(t) = I_m \sin(\omega t + \varphi_i) \longrightarrow \bar{I} = I_m \angle \varphi_i$$

Ex:



— x —

$$v(t) \longrightarrow \underbrace{\bar{V}}_{\text{Phasor associé à } v(t)}$$

$$\frac{dv}{dt} \longrightarrow \underbrace{\bar{V} \times (j\omega)}_{\text{Phasor associé à } \frac{dv}{dt}}$$

$$\int v\,dt \longrightarrow \underbrace{\frac{\bar{V}}{j\omega}}_{\text{Phasor associé à } \int v\,dt}$$

— x —

Phasors pour les éléments : R, L et C :

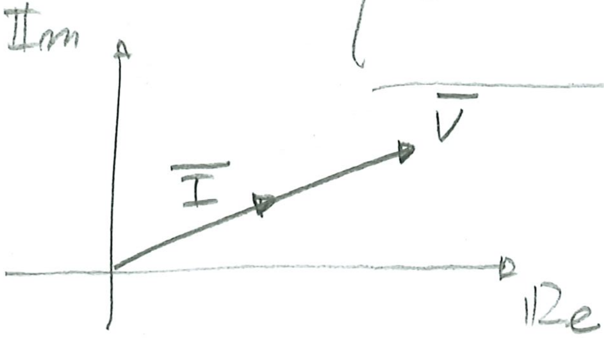
Résistance :

$$v(t) = R \cdot i(t) \longrightarrow \boxed{\overline{V} = R \cdot \overline{I}}$$

ou

$$\boxed{\overline{I} = \frac{\overline{V}}{R}}$$

$\boxed{\text{Le courant et la tension sont en phase}}$



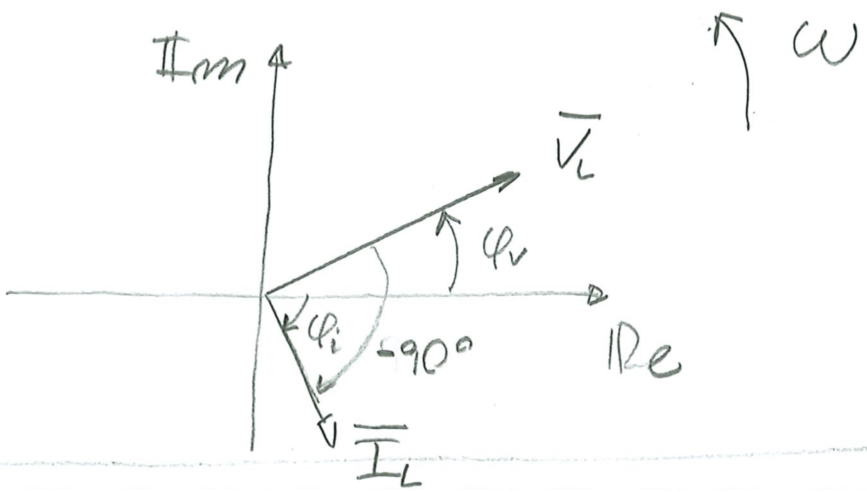
$$\boxed{\text{Si } v(t) = V_m \sin(\omega t + \varphi) \Longrightarrow i(t) = \frac{V_m}{R} \cdot \sin(\omega t + \varphi)}$$

— × —

Inductance :

$$v_L(t) = L \cdot \frac{d\,i_L(t)}{dt} \longrightarrow \overline{V_L} = L \cdot \overline{I_L} \times (j\omega)$$

$$\underbrace{\overline{V_L}}_{\text{Volt}} = \underbrace{(j\omega L)}_{\text{Ohm}(\Omega)} \cdot \underbrace{\overline{I_L}}_{\text{Ampère}}$$



Sur une inductance, le **courant** est **en retard** de 90° par rapport à la **tension**.

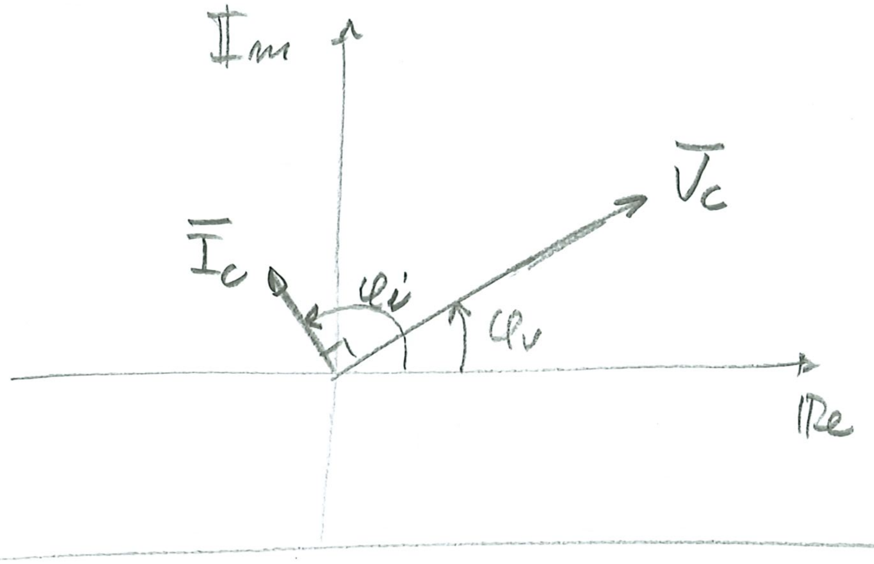
— x —

Condensateur :

$$i_c(t) = C\frac{dv_c(t)}{dt} \longrightarrow \overline{I_c} = C \cdot \overline{V_c} \cdot (j\omega)$$

$$\downarrow$$

$$\underbrace{\overline{V_c}}_{\text{Volt}} = \underbrace{\left(\frac{1}{j\omega C}\right)}_{\text{Ohm}(\Omega)} \underbrace{\overline{I_c}}_{\text{Ampère}}$$



Sur un condensateur, le courant est en avance de 90° par rapport à la tension

— × —

Résumé

	Élement	Domaine du temps	Phasor
	R	$v = R\,i$	$\overline{V} = R\,\overline{I}$
	L	$v = L\,\frac{di}{dt}$	$\overline{V} = j\omega L . \overline{I}$
	C	$i = C . \frac{dv}{dt}$	$\overline{V} = \frac{1}{\omega C} . \overline{I}$

Impédance complexe Z

Définition :

$$\boxed{Z = \frac{\bar{V}}{\bar{I}}}$$

Rapport entre le phasor tension et le phasor courant, mesurée en Ω.

Nombre complexe !

$$Z = \underbrace{R}_{\text{Résistance}} + j\underbrace{X}_{\text{Réactance}}$$

Résistance

$R \geq 0$

Réactance (sans la "j")

$\underbrace{X > 0}$ ou $\underbrace{X < 0}$

$X < 0$: impédance "capacitive"

$X > 0$: impédance "inductive"

$$\boxed{Z = R + jX = |Z| \angle \theta}$$

$$\boxed{\begin{aligned} |Z| &= \sqrt{R^2 + X^2} \\ \theta &= \operatorname{arctg}\left(\frac{X}{R}\right) \end{aligned}}$$

Voir résumé d'impédance !

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Impedance

$$Z = R + j0 = R\angle 0^\circ$$

$$Z = 0 + j\omega L = \omega L\angle +90^\circ$$

$$Z = 0 - \frac{1}{\omega C} = \frac{1}{\omega C}\angle -90^\circ$$

Il est parfois commode de travailler avec l'inverse de Z $\Longrightarrow$ <u>Admitance</u>

$$\boxed{Y = \frac{1}{\overline{Z}} = \frac{\overline{I}}{\overline{V}}}$$

$$Y = \underbrace{G}_{\text{Conductance (Siemens} = \frac{1}{\Omega})} + j\underbrace{B}_{\text{Susceptance (Siemens} = \frac{1}{\Omega})}$$

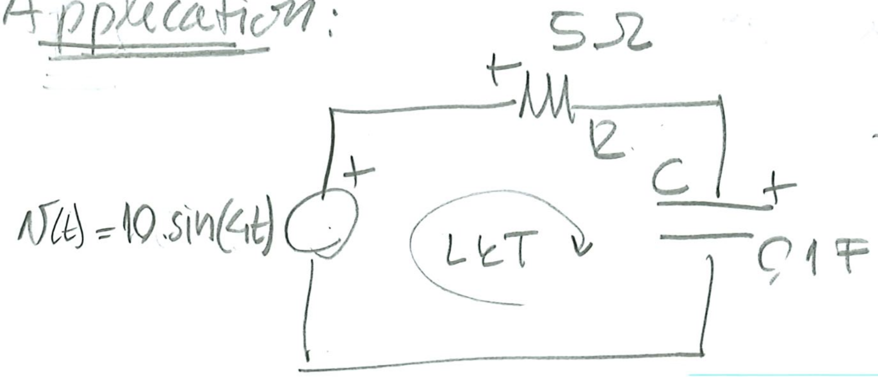
$$\boxed{G + jB = \frac{1}{R + jX}} \cdot \frac{R + jX}{R + jX}$$

Transformations:

$$\boxed{G = \frac{R}{R^2 + X^2}}, \quad \boxed{B = \frac{-X}{R^2 + X^2}}$$

Application :

Calculer $i(t)$

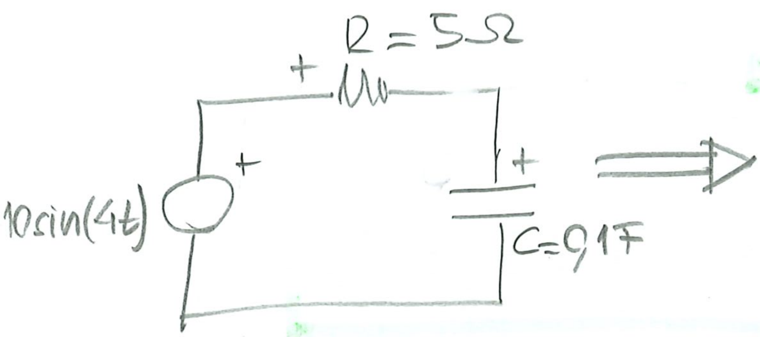


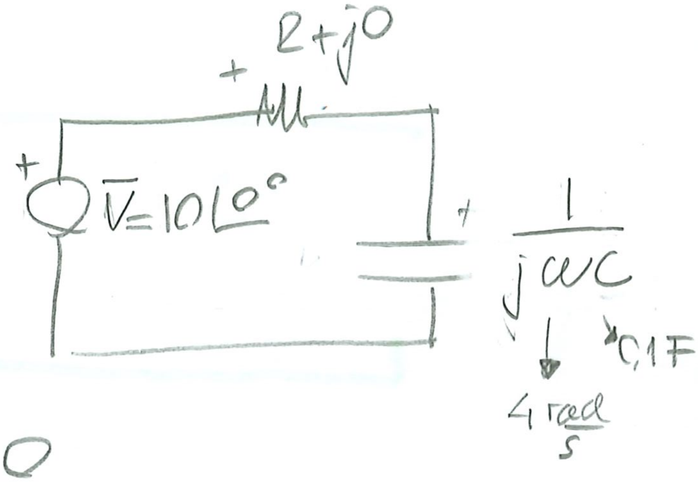
Lois de Kirchhoff avec des phasors :

LKT : $v_1(t) + v_2(t) + \dots + v_m(t) = 0 \iff \bar{V}_1 + \bar{V}_2 + \dots + \bar{V}_m = \underbrace{\bar{0}}_{0+j0}$

LKC : $i_1(t) + i_2(t) + \dots + i_m(t) = 0 \iff \bar{I}_1 + \bar{I}_2 + \dots + \bar{I}_m = \underbrace{\bar{0}}_{0+j0}$

Donc :





$\Rightarrow$ LKT : $-\bar{V} + \underbrace{\bar{V}_R}_{\underbrace{Z_R}_{R}\cdot\bar{I}} + \underbrace{\bar{V}_C}_{\underbrace{Z_C}_{\frac{1}{j\omega C}}\cdot\bar{V}} = 0$

Domc

$$\bar{I} \cdot R + \frac{\bar{I}}{j\omega C} = \bar{V}$$

$$\bar{I}\left(R + \frac{1}{j\omega C}\right) = \bar{V}$$

$$\Rightarrow \bar{I} = \frac{\bar{V}}{R + \frac{1}{j\omega C}} = \frac{\bar{V}}{R - j\frac{1}{\omega C}} = \frac{10 \angle 0^\circ}{5 - j2{,}5}$$

$$= \frac{10 \angle 0^\circ}{\sqrt{5^2 + 2{,}5^2} \angle \operatorname{arctg}\left(\frac{-2{,}5}{5}\right)} = \frac{10 \angle 0^\circ}{5{,}59 \angle -26{,}57^\circ}$$

$$= 1{,}78 \angle 26{,}57^\circ \ A$$

$$\Rightarrow i(t) = 1{,}78 \cdot \sin(4t + 26{,}57^\circ)$$

— x —

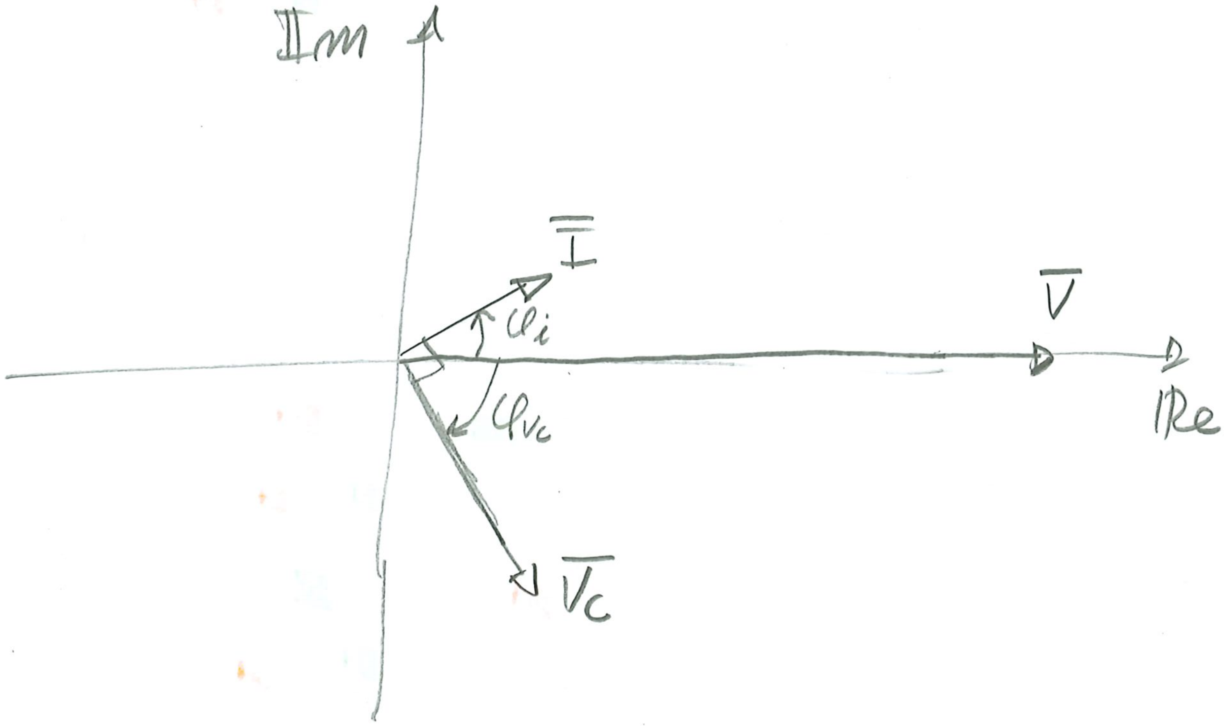
$v_C(t)$? , $v_R(t)$?

$$\bar{V}_C = \bar{I} \cdot Z_C = \bar{I} \times \frac{1}{j\omega C} = \bar{I} \left(0 - j\frac{1}{\omega C}\right)$$

$$= \bar{I} \times \frac{1}{\omega C} \angle -90^\circ = 1{,}78 \angle 26{,}57^\circ \times 2{,}5 \angle -90^\circ$$

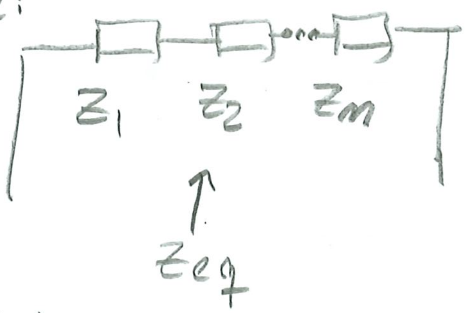
$$\overline{V_c} = (1{,}78 \times 2{,}5)\ \angle 26{,}57 + (-90°) = 4{,}45\ \angle -63{,}43$$

$$\Longrightarrow v_c(t) = 4{,}45 \sin(4t - 63{,}43°)$$



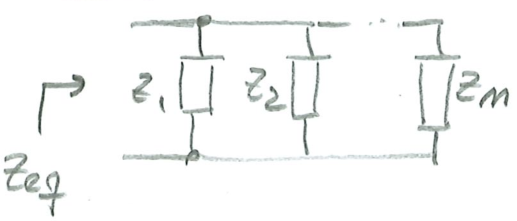
Association d'impédances

Série :



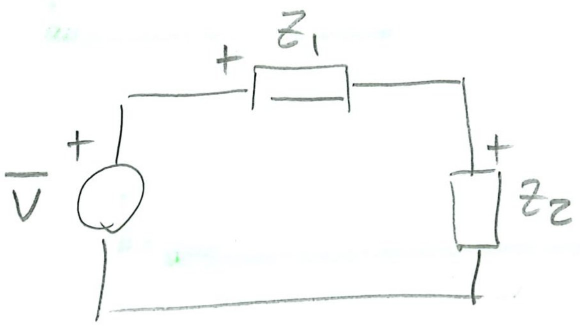
$$\Longrightarrow Z_{eq} = Z_1 + Z_2 + \dots + Z_n$$

Parallèle :



$$\Longrightarrow Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}}$$

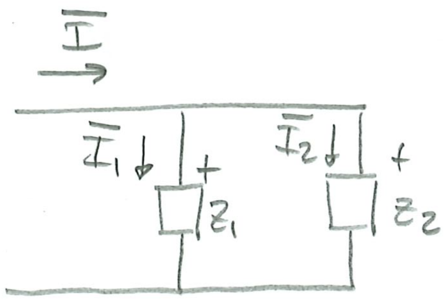
Diviseur de tension :



$$\overline{V}_1 = \frac{Z_1}{Z_1+Z_2} \cdot \overline{V}$$

$$\overline{V}_2 = \frac{Z_2}{Z_1+Z_2} \cdot \overline{V}$$

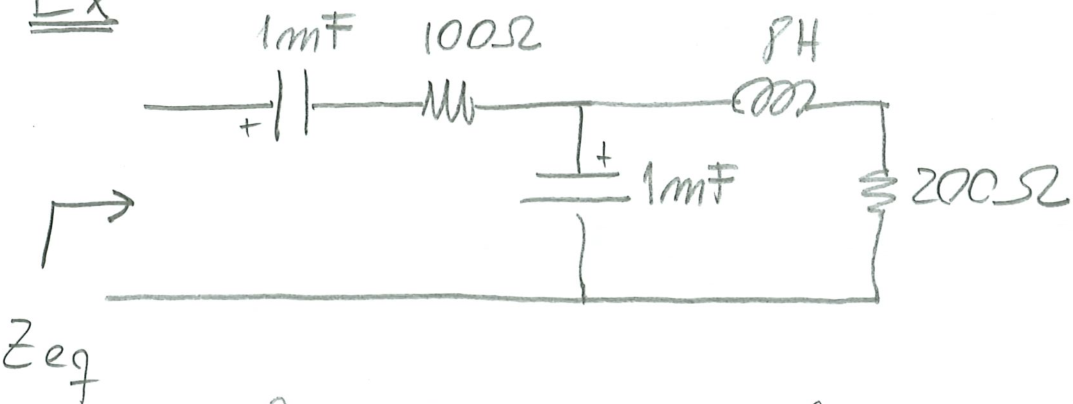
Diviseur de courant :



$$\overline{I}_1 = \frac{Z_2}{Z_1+Z_2} \cdot \overline{I}$$

$$\overline{I}_2 = \frac{Z_1}{Z_1+Z_2} \cdot \overline{I}$$

Ex



Calculer Z_{eq} (Réponse : $(149{,}52 - j195)\,\Omega$)